# HOMEWORK 2 – ANSWERS TO (MOST) PROBLEMS

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## Section 1.6: Inverse functions and logarithms

**1.6.3.** No; For example, even though $2 \neq 6$, $f(2) = f(6) = 2$

**1.6.5.** No (by the horizontal line test)

**1.6.17.** 0 (You want to find $x$ such that $g(x) = 4$, that is, find $x$ such that $x + e^x = 1$. Here, just guess!)

**1.6.18.**

(a) By the horizontal line test
(b) Domain of $f^{-1}$ = Range of $f = [-1, 3]$; Range of $f^{-1}$ = Domain of $f = [-3, 3]$
(c) 0
(d) $\approx -1.8$

**1.6.52.**

(a) $x = \pm\sqrt{1 + e^3}$
(b) $x = 0, \ln(2)$ (Let $X = e^x$ and solve the equation $X^2 - 3X + 2 = 0$ (by using the quadratic formula), then solve for $x$ using $e^x = X$)

**1.6.63.**

(a) $\frac{\pi}{3}$
(b) $\pi$

**1.6.69.** (Not on your problem set, but it's still an important problem. Also, 1.6.70 uses it) If $\theta = \sin^{-1}(x)$, then $\sin(\theta) = x$, then draw a triangle with hypothenuse 1, and opposite side x, and then the adjacent side becomes $\sqrt{1 - x^2}$, and so our answer becomes:

$$\cos(\sin^{-1}(x)) = \cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}$$

See the handout "Proof of the derivative of arccos" for a similar problem; Or look at your notes taken in section!

**1.6.70.** $\tan(\sin^{-1} x) = \frac{\sin(\sin^{-1}(x))}{\cos(\sin^{-1}(x))} = \frac{x}{\sqrt{1-x^2}}$ by the result of number 69!

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## Section 2.2: The limit of a function

**2.2.2.** If x approaches 1 from the left, then $f(x)$ approaches 3; If x approaches 1 from the right, then $f(x)$ approaches 7. No, left-hand-limits and right-hand-limits must be equal!

**2.2.6.**

(a) 4
(b) 4
(c) 4
(d) Undefined
(e) 1
(f) -1
(g) Does not exist (left and right-side limits not equal)
(h) 1
(i) 2
(j) Undefined
(k) 3
(l) Does not exist ($h$ does not approach one fixed value as x approaches 5 from the left)

**2.2.32.** $-\infty$ (numerator approaches $e^{-5} > 0$ while denominator approaches $0^-$

**2.2.33.** $-\infty$ ($x^2 - 9$ approaches $0^+$ and $\ln(0^+) = -\infty$

**2.2.46.** The mass blows up to $\infty$ ($\frac{v^2}{c^2}$ goes to $1^-$, so the denominator of the fraction goes to $0^+$, and so the whole fraction goes to $\infty$)

## Section 2.3: Calculating limits using the limit laws

**2.3.9.** Just plug in $x = 2$

**2.3.18.** 12 (Use the formula $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$)

**2.3.29.** $\frac{1}{2}$ (put under a common denominator and multiply by the conjugate form)

**2.3.32.** $-\frac{2}{x^3}$ (put under a common denominator and expand the numerator out)

**2.3.37.** 7 (use the squeeze theorem)

**2.3.40.** 0 (by squeeze theorem, because $-1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1$)

**2.3.60.** Let $a = 0$ and $f(x) = \sin\left(\frac{1}{x}\right)$ (or $\frac{1}{x}$), and $g(x) = -f(x)$.

**2.3.61.** Let $a = 0$ and $f(x) = \sin\left(\frac{1}{x}\right)$ (or $\frac{1}{x}$), and $g(x) = \frac{1}{f(x)}$

**2.3.64. Hints:** Use the following steps:

(a) Find the coordinates of $Q$. For this, solve for $x$ and $y$ in the system of equations:

$$\begin{cases} (x-1)^2 + y^2 = 1 \\ x^2 + y^2 = r^2 \end{cases}$$

For this, plug in $y^2 = r^2 - x^2$ in the first equation and solve for $x$, then solve for $y$ in $y^2 = r^2 - x^2$; remember that you want $x > 0$ and $y > 0$, according to the picture). The answer gives you the coordinates of $Q$

(b) Now that you know the coordinates of $P$ and $Q$, find the equation of the line going through $P$ and $Q$

(c) Find the $x$-intercept of that line (set $y = 0$ and solve for $x$)

(d) Finally, take the limit as $r \to 0^+$ of the answer you found in ($c$). To do this, multiply as usual by the conjugate form.

**Answers:**

(a) $Q = (\frac{r^2}{2}, r\sqrt{1 - \frac{r^2}{4}})$

(b) $y = \frac{2}{r}\left(\sqrt{1 - \frac{r^2}{4}} - 1\right)x + r$

(c) $x-$ intercept $= \frac{r^2}{2\left(1 - \sqrt{1 - \frac{r^2}{4}}\right)}$

(d) $\boxed{4}$